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**JEE
MAIN
Sept.
2020**

QUESTION PAPER WITH SOLUTION

MATHEMATICS _ 4 Sep. _ SHIFT - 1



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1. Let $y=y(x)$ be the solution of the differential equation, $xy'-y=x^2(x\cos x+\sin x)$, $x > 0$. if $y(\pi) = \pi$, then

$y''\left(\frac{\pi}{2}\right) + y\left(\frac{\pi}{2}\right)$ is equal to

- (1) $2 + \frac{\pi}{2} + \frac{\pi^2}{4}$ (2) $2 + \frac{\pi}{2}$ (3) $1 + \frac{\pi}{2}$ (4) $1 + \frac{\pi}{2} + \frac{\pi^2}{4}$

Sol. (2)

$$xy' - y = x^2(x \cos x + \sin x) \quad x > 0, y(\pi) = \pi$$

$$y' - \frac{1}{x}y = x\{x\cos x + \sin x\}$$

$$\text{I.F.} = e^{-\int \frac{1}{x} dx} = e^{-\ln x} = \frac{1}{x}$$

$$\therefore y \cdot \frac{1}{x} = \int \frac{1}{x} \cdot x(x \cos x + \sin x) dx$$

$$\frac{y}{x} = \int (x \cos x + \sin x) dx$$

$$\frac{y}{x} = \int \frac{d}{dx}(x \sin x) dx$$

$$\frac{y}{x} = x \sin x + C$$

$$\Rightarrow y = x^2 \sin x + cx$$

$$x = \pi, y = \pi$$

$$\pi = \pi C \Rightarrow C = 1$$

$$y = x^2 \sin x + x \Rightarrow y\left(\frac{\pi}{2}\right) = \frac{\pi^2}{4} + \frac{\pi}{2}$$

$$y' = 2x \sin x + x^2 \cos x + 1$$

$$y'' = 2 \sin x + 2x \cos x + 2x \cos x - x^2 \sin x$$

$$y''\left(\frac{\pi}{2}\right) = 2 - \frac{\pi^2}{4} \Rightarrow y\left(\frac{\pi}{2}\right) + y''\left(\frac{\pi}{2}\right) = 2 + \frac{\pi}{2}$$

2. The value of $\sum_{r=0}^{20} {}^{50-r}C_6$ is equal to:

- (1) ${}^{51}C_7 - {}^{30}C_7$ (2) ${}^{51}C_7 + {}^{30}C_7$ (3) ${}^{50}C_7 - {}^{30}C_7$ (4) ${}^{50}C_6 - {}^{30}C_6$

Sol. (1)

$$\sum_{r=0}^{20} {}^{50-r}C_6$$

$$\Rightarrow {}^{50}C_6 + {}^{49}C_6 + {}^{48}C_6 + \dots + {}^{31}C_6 + {}^{30}C_6$$

add and subtract ${}^{30}C_7$

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Using

$${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r \Rightarrow {}^{30}C_6 + {}^{30}C_7 = {}^{31}C_7$$

$${}^{31}C_6 + {}^{31}C_7 = {}^{32}C_7$$

Similarly solving

$${}^{51}C_7 - {}^{30}C_7$$

3. Let $[t]$ denote the greatest integer $\leq t$. Then the equation in $x, [x]^2 + 2[x + 2] - 7 = 0$ has :
 (1) exactly four integral solutions. (2) infinitely many solutions.
 (3) no integral solution. (4) exactly two solutions.

Sol.

(2)

$$[x]^2 + 2[x + 2] - 7 = 0$$

$$[x]^2 + 2[x] - 3 = 0$$

$$\text{let } [x] = y$$

$$y^2 + 3y - y - 3 = 0$$

$$(y - 1)(y + 3) = 0$$

$$[x] = 1 \text{ or } [x] = -3$$

$$x \in [1, 2) \quad \& \quad x \in [-3, -2)$$

4. Let $P(3,3)$ be a point on the hyperbola, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal to it at P intersects the x -axis at $(9,0)$ and e is its eccentricity, then the ordered pair (a^2, e^2) is equal to :

(1) $(9,3)$

(2) $\left(\frac{9}{2}, 2\right)$

(3) $\left(\frac{9}{2}, 3\right)$

(4) $\left(\frac{3}{2}, 2\right)$

Sol.

(3)

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$P(3,3)$

$$\frac{9}{a^2} - \frac{9}{b^2} = 1$$

....(1)

$$\text{Equation of normal} \Rightarrow \frac{a^2x}{3} + \frac{b^2y}{3} = a^2e^2$$

$$\text{at } x\text{-axis} \Rightarrow y = 0$$

$$\frac{a^2x}{3} = a^2e^2 \Rightarrow x = 3e^2 = 9$$

$$e^2 = 3$$

$$e = \sqrt{3}$$

$$e^2 = 1 + \frac{b^2}{a^2} = 3$$

$$b^2 = 2a^2$$

...(2)

put in equation 1

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$$\frac{9}{a^2} - \frac{9}{2a^2} = 1 \Rightarrow \frac{9}{2a^2} = 1 \Rightarrow a^2 = \frac{9}{2}$$

$$\therefore (a^2, e^2) = \left(\frac{9}{2}, 3\right)$$

5. Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($a > b$) be a given ellipse, length of whose latus rectum is 10. If its eccentricity is the maximum value of the function, $\phi(t) = \frac{5}{12} + t - t^2$, then $a^2 + b^2$ is equal to
 (1) 135 (2) 116 (3) 126 (4) 145

Sol.

$$\text{L.R} = \frac{2b^2}{a} = 10 \quad \dots(1)$$

$$\phi(t) = \frac{5}{12} - \left(t - \frac{1}{2}\right)^2 + \frac{1}{4} = \frac{8}{12} - \left(t - \frac{1}{2}\right)^2$$

$$\therefore \phi(t)_{\max} = \frac{2}{3} = e$$

$$e^2 = 1 - \frac{b^2}{a^2} = \frac{4}{9} \Rightarrow \frac{b^2}{a^2} = \frac{5}{9}$$

$$\frac{b^2}{a \cdot a} = \frac{5}{9} \text{ from (1)}$$

$$\frac{5}{a} = \frac{5}{9} \Rightarrow a = 9$$

$$\therefore b^2 = 45$$

$$a^2 + b^2 = 45 + 81 = 126$$

6. Let $f(x) = \int \frac{\sqrt{x}}{(1+x)^2} dx$ ($x \geq 0$). Then $f(3) - f(1)$ is equal to :

- (1) $-\frac{\pi}{6} + \frac{1}{2} + \frac{\sqrt{3}}{4}$ (2) $\frac{\pi}{6} + \frac{1}{2} - \frac{\sqrt{3}}{4}$ (3) $-\frac{\pi}{12} + \frac{1}{2} + \frac{\sqrt{3}}{4}$ (4) $\frac{\pi}{12} + \frac{1}{2} - \frac{\sqrt{3}}{4}$

Sol.

$$f(x) = \int \frac{\sqrt{x}}{(1+x)^2} dx$$

$$x = \tan^2 t$$

$$dx = 2 \tan t \sec^2 t dt$$

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$$f(x) = \int \frac{\tan t \cdot 2 \tan t \sec^2 t dt}{\sec^4 t}$$

$$= 2 \int \sin^2 t dt$$

$$x = 3 \Rightarrow t = \frac{\pi}{3}$$

$$x = 1 \Rightarrow t = \frac{\pi}{4}$$

$$\therefore f(3) - f(1) = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (1 - \cos 2t) dt \Rightarrow \left(t - \frac{1}{2} \sin 2t \right)_{\frac{\pi}{4}}^{\frac{\pi}{3}} = \frac{\pi}{12} + \frac{1}{2} - \frac{\sqrt{3}}{4}$$

7. If $1 + (1 - 2^2 \cdot 1) + (1 - 4^2 \cdot 3) + (1 - 6^2 \cdot 5) + \dots + (1 - 20^2 \cdot 19) = \alpha - 220\beta$, then an ordered pair (α, β) is equal to:
 (1) (10,97) (2) (11,103) (3) (11,97) (4) (10,103)

Sol.

$$(2) \quad 1 + S_n$$

$$T_n = 1 - (2n)^2(2n - 1)$$

$$= 1 - 4n^2(2n - 1)$$

$$= 1 - 8n^3 + 4n^2$$

$$S_n = \sum_{n=1}^{10} T_n = n - \sum 8n^3 + \sum 4n^2$$

$$= n - 8 \times \frac{n^2 (n+1)^2}{4} + \frac{4n(n+1)(2n+1)}{6}$$

$$= 10 - 2 \times 100 \times 121 + \frac{2}{3} \times 10 \times 11 \times 21$$

$$= 10 - 24200 + 1540$$

$$= 10 - 22660$$

$$\therefore \text{Sum of series} = 11 - 22660 = \alpha - 220\beta$$

$$\alpha = 11, \beta = 103$$

8. The integral $\int \left(\frac{x}{x \sin x + \cos x} \right)^2 dx$ is equal to

(where C is a constant of integration):

(1) $\tan x - \frac{x \sec x}{x \sin x + \cos x} + C$

(2) $\sec x - \frac{x \tan x}{x \sin x + \cos x} + C$

(3) $\sec x + \frac{x \tan x}{x \sin x + \cos x} + C$

(4) $\tan x + \frac{x \sec x}{x \sin x + \cos x} + C$

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Sol. (1)

$$\int \left(\frac{x}{x \sin x + \cos x} \right)^2 dx$$

$$\int \underbrace{x \sec x}_I \cdot \underbrace{\frac{x \cos x}{(x \sin x + \cos x)^2}}_{II} dx$$

$$x \sec x \left(\frac{-1}{x \sin x + \cos x} \right) + \int \frac{\sec x + x \sec x \tan x}{(x \sin x + \cos x)} dx$$

$$\Rightarrow \frac{-x \sec x}{x \sin x + \cos x} + \int \frac{(\cos x + x \sin x)}{\cos^2 x (x \sin x + \cos x)} dx \Rightarrow \frac{-x \sec x}{x \sin x + \cos x} + \tan x + C$$

9. Let $f(x) = |x-2|$ and $g(x) = f(f(x))$, $x \in [0,4]$. Then $\int_0^3 (g(x) - f(x)) dx$ is equal to:

(1) $\frac{1}{2}$

(2) 0

(3) 1

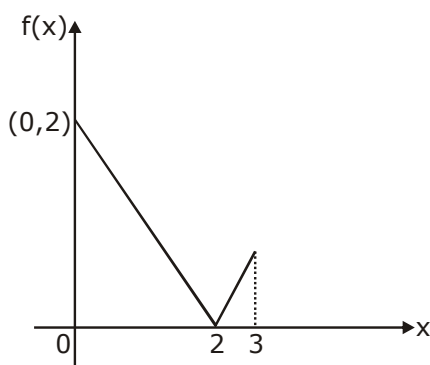
(4) $\frac{3}{2}$

Sol. (3)

$$f(x) = |x - 2|$$

$$g(x) = ||x - 2| - 2| = \begin{cases} \text{if } x \geq 2 & \Rightarrow |x - 4| \\ \text{if } x < 2 & \Rightarrow |-x| \end{cases}$$

$$\therefore \int_0^3 (g(x) - f(x)) dx$$



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$$\begin{aligned}
 &= \int_0^3 g(x) - \int_0^3 f(x) dx \\
 &= \int_0^2 x dx + \int_2^3 (4-x) dx - \int_0^2 (2-x) dx - \int_2^3 (x-2) dx \\
 &\Rightarrow \left(\frac{x^2}{2}\right)_0^2 + \left(4x - \frac{x^2}{2}\right)_2^3 + \left(\frac{x^2}{2} - 2x\right)_0^2 - \left(\frac{x^2}{2} - 2x\right)_2^3 \\
 &\Rightarrow 2 + \left\{12 - \frac{9}{2} - 8 + 2\right\} + \{2 - 4\} - \left(\frac{9}{2} - 6 - 2 + 4\right) \\
 &= 2 + \left\{6 - \frac{9}{2}\right\} - 2 - \left\{\frac{9}{2} - 4\right\} = 2 + \frac{3}{2} - \left(2 + \frac{1}{2}\right) = \frac{7}{2} - \frac{5}{2} = 1
 \end{aligned}$$

10. Let x_0 be the point of Local maxima of $f(x) = \vec{a} \cdot (\vec{b} \times \vec{c})$, where $\vec{a} = x\hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = -2\hat{i} + x\hat{j} - \hat{k}$ and $\vec{c} = 7\hat{i} - 2\hat{j} + x\hat{k}$. Then the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ at $x=x_0$ is :
- (1) -22 (2) -4 (3) -30 (4) 14

Sol. (1)

$$\begin{aligned}
 \vec{a} \cdot (\vec{b} \times \vec{c}) &= \begin{vmatrix} x & -2 & 3 \\ -2 & x & -1 \\ 7 & -2 & x \end{vmatrix} \\
 &\Rightarrow x\{x^2 - 2\} + 2\{-2x + 7\} + 3\{4 - 7x\} \\
 &= x^3 - 2x - 4x + 14 + 12 - 21x \\
 f(x) &= x^3 - 27x + 26 \\
 f'(x) &= 3x^2 - 27 = 0 \Rightarrow x = \pm 3 \\
 \text{Max at } x_0 &= -3 \\
 \therefore \vec{a} &= (-3, -2, 3), \vec{b} = (-2, -3, -1), \vec{c} = (7, -2, -3) \\
 \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} &= 6 + 6 - 3 - 14 + 6 + 3 - 21 + 4 - 9 \\
 &= 25 - 47 = -22
 \end{aligned}$$

11. A triangle ABC lying in the first quadrant has two vertices as A(1,2) and B(3,1) If $\angle BAC = 90^\circ$, and $\text{ar}(\triangle ABC) = 5\sqrt{5}$ s units, then the abscissa of the vertex C is :
- (1) $1 + \sqrt{5}$ (2) $1 + 2\sqrt{5}$ (3) $2\sqrt{5} - 1$ (4) $2 + \sqrt{5}$

Sol. (2)

$$AB = \sqrt{4+1} = \sqrt{5}$$

$$\frac{1}{2} \times \sqrt{5} \times x = 5\sqrt{5}$$

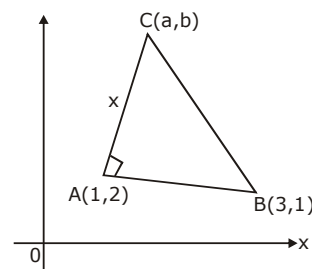
$$x = 10$$

$$m_{AB} = \frac{1}{-2}$$

$$m_{AC} = 2 = \tan\theta$$

$$\therefore \sin\theta = \frac{2}{\sqrt{5}}, \cos\theta = \frac{1}{\sqrt{5}}$$

$$\text{by parametric co-ordinates } a = 1 + 10 \times \frac{1}{\sqrt{5}} = 1 + 2\sqrt{5}$$



12. Let f be a twice differentiable function on $(1,6)$. If $f(2)=8$, $f'(2)=5$, $f'(x) \geq 1$ and $f''(x) \geq 4$, for all $x \in (1,6)$, then:

(1) $f(5)+f'(5) \geq 28$

(2) $f'(5)+f''(5) \leq 20$

(3) $f(5) \leq 10$

(4) $f(5)+f'(5) \leq 26$

Sol. (1)

$$f(2) = 8, f'(2) = 5, f'(x) \geq 1, f''(x) \geq 4$$

$$x \in (1,6)$$

$$\int_2^5 f'(x) dx \geq \int_2^5 1 dx$$

$$f(5) - f(2) \geq 3$$

$$f(5) \geq 11 \quad \dots(1)$$

$$\text{also } \int_2^5 f''(x) dx \geq \int_2^5 4 dx$$

$$f'(5) - f'(2) \geq 12$$

$$f'(5) \geq 17$$

$$\dots(2)$$

13. Let α and β be the roots of $x^2-3x+p=0$ and γ and δ be the roots of $x^2-6x+q=0$. If $\alpha, \beta, \gamma, \delta$ form a geometric progression. Then ratio $(2q+p) : (2q-p)$ is:

(1) 33 : 31

(2) 9 : 7

(3) 3 : 1

(4) 5 : 3

Sol. (2)

$$x^2 - 3x + p = 0 (\alpha, \beta)$$

$$x^2 - 6x + q = 0 (\gamma, \delta)$$

$$\alpha + \beta = 3$$

$$\gamma + \delta = 6$$

$$\alpha = a, \beta = ar, \gamma = ar^2, \delta = ar^3$$

$$a(1+r) = 3 \quad \dots(1)$$

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$$ar^2(1+r) = 6 \quad \dots(2)$$

Divide (2) by (1)

$$r^2 = 2, r = \sqrt{2} \Rightarrow a = \frac{3}{\sqrt{2}+1}$$

$$\alpha = \frac{3}{\sqrt{2}+1}, \beta = \frac{3\sqrt{2}}{\sqrt{2}+1}, \gamma = \frac{3.2}{\sqrt{2}+1}, \delta = \frac{3.2\sqrt{2}}{\sqrt{2}+1}$$

$$\alpha\beta = p = \frac{9\sqrt{2}}{(\sqrt{2}+1)^2}, \gamma\delta = \frac{36\sqrt{2}}{(\sqrt{2}+1)^2} \Rightarrow \frac{72+9}{72-9} = \frac{81}{63}$$

$$= 9/7$$

- 14.** Let $u = \frac{2z+i}{z-ki}$, $z = x + iy$ and $k > 0$. If the curve represented by $\text{Re}(u) + \text{Im}(u) = 1$ intersects the y-axis at the points P and Q where $PQ = 5$, then the value of k is :
- (1) 4 (2) 1/2 (3) 2 (4) 3/2

Sol. (3)

$$u = \frac{2z+i}{z-ki}, \quad z = x + iy$$

$$= \frac{2x+i(2y+1)}{x+i(y-k)} \times \frac{x-i(y-k)}{x-i(y-k)}$$

$$\Rightarrow \frac{2x^2 + (2y+1)(y-k) + i\{2xy + x - 2xy + 2xk\}}{x^2 + (y-k)^2}$$

$$\text{Re}(u) + \text{Im}(u) = 1$$

$$2x^2 + (2y+1)(y-k) + x + 2xk = x^2 + (y-k)^2$$

at y - axis, $x = 0$

$$(2y+1)(y-k) = (y-k)^2$$

$$2y^2 + y - 2yk - k = y^2 + k^2 - 2yk$$

$$y^2 + y - (k+k^2) = 0 \quad (y_1, y_2)$$

diff. of roots = 5

$$\sqrt{1 + 4k + 4k^2} = 5$$

$$4k^2 + 4k = 24$$

$$k^2 + k - 6 = 0$$

$$(k+3)(k-2) = 0$$

$$k = 2$$

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15. If $A = \begin{bmatrix} \cos \theta & i \sin \theta \\ i \sin \theta & \cos \theta \end{bmatrix}$, $\left(\theta = \frac{\pi}{24}\right)$ and $A^5 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where $i = \sqrt{-1}$, then which one of the following is not true?

(1) $a^2 - d^2 = 0$ (B) $a^2 - c^2 = 1$ (C) $0 \leq a^2 + b^2 \leq 1$ (D) $a^2 - b^2 = \frac{1}{2}$

Sol. (4)

$$\begin{bmatrix} c & is \\ is & c \end{bmatrix} \begin{bmatrix} c & is \\ is & c \end{bmatrix} = z \begin{bmatrix} c^2 - s^2 & 2ics \\ 2ics & c^2 - s^2 \end{bmatrix} = \begin{bmatrix} \cos 2\theta & i \sin 2\theta \\ i \sin 2\theta & \cos 2\theta \end{bmatrix} \quad (\text{where } c = \cos \theta, s = \sin \theta)$$

$$A^5 = \begin{bmatrix} \cos(2^4\theta) & i \sin(2^4\theta) \\ i \sin(2^4\theta) & \cos(2^4\theta) \end{bmatrix}$$

$$a = d = \cos(16\theta)$$

$$b = c = i \sin(16\theta)$$

$$a^2 - b^2 = \cos^2(16\theta) + \sin^2 16\theta = 1$$

16. The mean and variance of 8 observations are 10 and 13.5, respectively. If 6 of these observations are 5, 7, 10, 12, 14, 15, then the absolute difference of the remaining two observations is:

(1) 3 (2) 9 (3) 7 (4) 5

Sol. 3

$$\frac{5 + 7 + 10 + 12 + 14 + 15 + x + y}{8} = 10$$

$$x + y = 17 \quad \dots(1)$$

$$\text{variance} = \frac{739 + x^2 + y^2}{8} - 100 = 13.5$$

$$x^2 + y^2 = 169 \quad \dots(2)$$

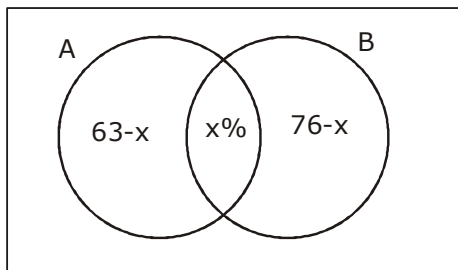
$$\therefore x = 12, y = 5$$

$$|x - y| = 7$$

17. A survey shows that 63% of the people in a city read newspaper A whereas 76% read newspaper B. If x% of the people read both the newspapers, then a possible value of x can be:

(1) 37 (2) 29 (3) 65 (4) 55

Sol. (4)



$A \cup B = 13 - x \leq 100$
 $x \geq 39$
 also $x \leq 63$

18. Given the following two statements:

(S₁): $(q \vee p) \rightarrow (P \leftrightarrow \sim q)$ is a tautology

(S₂): $\sim q \wedge (\sim p \leftrightarrow q)$ is a fallacy. Then:

(1) only (S₁) is correct.

(2) both (S₁) and (S₂) are correct.

(3) only (S₂) is correct

(4) both (S₁) and (S₂) are not correct.

Sol. (4)

	p	q	$\sim q$	$q \vee p$	$p \leftrightarrow \sim q$	$(q \vee p) \rightarrow (p \leftrightarrow \sim q)$
	T	T	F	T	F	F
	T	F	T	T	T	T
S ₁ =	F	T	F	T	T	T
	F	F	T	F	F	T

S₁ is not correct

	p	q	$\sim q$	$\sim p$	$\sim p \leftrightarrow q$	$\sim q \wedge (\sim p \leftrightarrow q)$
	T	T	F	F	F	F
	T	F	T	F	T	T
S ₂ =	F	T	F	T	T	F
	F	F	T	T	F	F

S₂ is false

19. Two vertical poles AB=15 m and CD=10 m are standing apart on a horizontal ground with points A and C on the ground. If P is the point of intersection of BC and AD, then the height of P (in m) above the line AC is:

(1) 5

(2) 20/3

(3) 10/3

(4) 6

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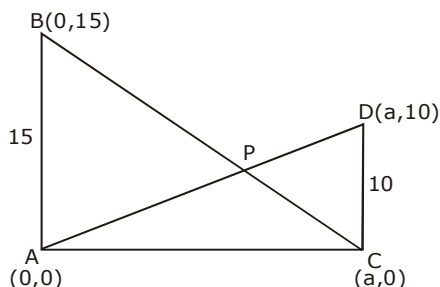
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Sol. (4)



equation of AD : $y = \frac{10x}{a}$

equation of BC : $\frac{x}{a} + \frac{y}{15} = 1$

$\Rightarrow \frac{a.y}{10a} + \frac{y}{15} = 1 \Rightarrow \frac{3y + 2y}{30} = 1$

$5y = 30 \Rightarrow y = 6$

20. If $(a + \sqrt{2} b \cos x)(a - \sqrt{2} b \cos y) = a^2 - b^2$, where $a > b > 0$, then $\frac{dx}{dy}$ at $(\frac{\pi}{4}, \frac{\pi}{4})$ is:

(1) $\frac{a+b}{a-b}$

(2) $\frac{a-2b}{a+2b}$

(3) $\frac{a-b}{a+b}$

(4) $\frac{2a+b}{2a-b}$

Sol. (1)

$(a + \sqrt{2} b \cos x)(a - \sqrt{2} b \cos y) = a^2 - b^2$

diff both sides w.r.t y

$-\sqrt{2} b \sin x \cdot \frac{dx}{dy} (a - \sqrt{2} b \cos y) + (a + \sqrt{2} b \cos x)(\sqrt{2} b \sin y) = 0$

$x = y = \frac{\pi}{4} \Rightarrow \frac{-b dx}{dy} (a - b) + (a + b)(b) = 0$

$\frac{dx}{dy} = \frac{a+b}{a-b}$

21. Suppose a differentiable function $f(x)$ satisfies the identity $f(x+y) = f(x) + f(y) + xy^2 + x^2y$, for all real x

and y . If $\lim_{x \rightarrow 0} \frac{f(x)}{x} = 1$, then $f'(3)$ is equal to.....

Sol. $f(x+y) = f(x) + f(y) + xy^2 + x^2y$
 $x = y = 0$
 $f(0) = 2f(0) \Rightarrow f(0) = 0$

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Partially diff. w.r.t. x
 $f'(x+y) = f'(x) + y^2 + 2xy$
 $x = 0, y = x$

$$f'(x) = f'(0) + x^2 \quad \text{given } \lim_{x \rightarrow 0} \frac{f(x)}{x} = 1$$

$$f'(x) = 1 + x^2 \quad \text{by L' hospital}$$

$$\therefore f(x) = x + \frac{x^3}{3} + c \quad \lim_{x \rightarrow 0} \frac{f'(x)}{1} = 1$$

$$\text{put } x = 0 \Rightarrow c = 0 \quad f'(0) = 1$$

$$f'(3) = 10$$

22. If the equation of a plane P, passing through the intersection of the planes, $x+4y-z+7=0$ and $3x+y+5z=8$ is $ax+by+6z=15$ for some $a, b \in \mathbb{R}$, then the distance of the point $(3, 2, -1)$ from the plane P is.....

Sol. $p_1 + \lambda p_2 = 0$
 $(x + 4y - z + 7) + \lambda (3x + y + 5z - 8) = ax + by + 6z - 15$

$$\frac{1 - 3\lambda}{a} = \frac{4 + \lambda}{b} = \frac{-1 + 5\lambda}{6} = \frac{7 - 8\lambda}{-15}$$

$$\therefore 15 - 75\lambda = 42 - 48\lambda$$

$$-27 = 27\lambda$$

$$\lambda = -1$$

$$\therefore \text{plane is } -2x + 3y - 6z + 15 = 0$$

$$d = \left| \frac{-6 + 6 + 6 + 15}{\sqrt{4 + 9 + 36}} \right| = 3$$

23. If the system of equations

$$x - 2y + 3z = 9$$

$$2x + y + z = b$$

$$x - 7y + az = 24, \text{ has infinitely many solutions, then } a - b \text{ is equal to.....}$$

Sol. $D = 0$

$$\begin{vmatrix} 1 & -2 & 3 \\ 2 & 1 & 1 \\ 1 & -7 & a \end{vmatrix} = 0$$

$$1(a + 7) + 2(2a - 1) + 3(-14 - 1) = 0$$

$$a + 7 + 4a - 2 - 45 = 0$$

$$5a = 40$$

$$a = 8$$

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$$D_1 = \begin{vmatrix} 9 & -2 & 3 \\ b & 1 & 1 \\ 24 & -7 & 8 \end{vmatrix} = 0$$

$$\begin{aligned} \Rightarrow 9(8 + 7) + 2(8b - 24) + 3(-7b - 24) &= 0 \\ \Rightarrow 135 + 16b - 48 - 21b - 72 &= 0 \\ 15 = 5b \Rightarrow b &= 3 \\ a - b &= 5 \end{aligned}$$

24. Let $(2x^2 + 3x + 4)^{10} = \sum_{r=0}^{20} a_r x^r$. Then $\frac{a_7}{a_{13}}$ is equal to

Sol. 8

$$(2x^2 + 3x + 4)^{10} = \sum_{r=0}^{20} a_r x^r$$

a_7 = coeff of x^7

a_{13} = coeff of x^{13}

$$\frac{10!}{p!q!r!} (2x^2)^p (3x)^q (4)^r$$

for x^7

p	q	r
3	1	6
2	3	5
1	5	4
0	7	3

$$a_7 = \frac{2^3 \cdot 3 \cdot 10!}{3!6!} + \frac{10! \cdot 2^2 \cdot 3^3}{2!3!5!} + \frac{10! \cdot 2 \cdot 3^5}{5!4!} + \frac{10! \cdot 3^7}{7!3!}$$

for x^{13}

p	q	r
6	1	3
5	3	2
4	5	1
3	7	0

$$a_{13} = \frac{2^6 \cdot 3 \cdot 10!}{6!3!} + \frac{2^5 \cdot 3^3 \cdot 10!}{5!3!2!} + \frac{2^4 \cdot 3^5 \cdot 10!}{4!5!} + \frac{2^3 \cdot 10!}{3!7!} \therefore \frac{a_7}{a_{13}} = 8$$

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25. The probability of a man hitting a target is $\frac{1}{10}$. The least number of shots required, so that the probability of his hitting the target at least once is greater than $\frac{1}{4}$, is

Sol. 3

$$P(H) = \frac{1}{10} ; P(M) = \frac{9}{10}$$

$$P(H) + P(M). P(H) + P(M). P(M). P(H) + \dots\dots\dots$$

$$= 1 - P(M)^n \geq \frac{1}{4}$$

$$= 1 - \left(\frac{9}{10}\right)^n \geq \frac{1}{4}$$

$$\left(\frac{9}{10}\right)^n \leq \frac{3}{4} ; n \geq 3$$

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